

Constituent Quark Model Approach to Heavy Baryon Transitions

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ABSTRACT

I discuss the structure of current-induced bottom baryon to charm baryon transitions, and the structure of pion and photon transitions between heavy charm or bottom baryons in the Heavy Quark Symmetry (HQS) limit as $m_Q \rightarrow \infty$. By doing a spin-parity analysis I derive a general formula which allows one to enumerate the independent HQS amplitudes in the three types of transitions. I go on to show that use of the constituent quark model for the light-side diquark transitions leads to a considerable reduction in the number of independent amplitudes derived in the HQS limit. The discussion includes the ground state s -wave as well as the p -wave heavy baryons.

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1 Introduction

In heavy hadrons made up of a heavy quark and a light quark system the dynamics of the heavy side and the light side completely decouple in the heavy mass limit as $m_Q \rightarrow \infty$. Since the dynamics of the heavy quark is known the heavy quark can be viewed as providing a probe of the unknown light-side dynamics. For example, in current induced bottom to charm hadron transitions there is a $b \rightarrow c$ transition on the heavy side. The dynamics of this transition is completely specified. The light side knows nothing about the heavy-side transition except for a light-side velocity change. The velocity change is necessary since the light side quark system has to readjust its velocity to keep up with the emerging energetic c -quark in order to form the final hadron. There are also some trivial angular momentum factors that provide for the requisite projections on the initial and final heavy hadrons with their given spins and parities. In the heavy flavour conserving pion and photon transitions the pions and photons are emitted from the light side and the heavy side knows nothing about these light-side transitions. It is then an angular momentum coupling exercise to determine the number of independent amplitudes that describe the heavy hadron transitions in the HQS limit. As the dynamics of the heavy-side transitions is entirely known, all that is needed is to determine the structure of the light-side transitions.

Turning to heavy baryons made up of a heavy quark and a light diquark system one thus has to study the dynamics of the transitions between light diquark systems very much like one has been studying light “triquark” transitions in ordinary light baryon transitions. Many different approaches of varying degrees of sophistication have been developed for this purpose in the past. Among these is the constituent quark model approach which has been singularly succesful in the light baryon sector. Although one is far from being able to derive the constituent quark model from first principles one is now beginning to understand that the constituent quark model may in fact emerge in the large N_C limit [1], at least in the baryon sector. It is then very tempting and quite natural to try and get a first handle on the light diquark transitions in the heavy baryon sector by using the constituent quark model approach for these. The constituent quark model predictions will provide simple bench mark predictions for more sophisticated approaches such as QCD

sum rule and lattice calculations to be carried out at a later stage.

This defines the aim and the scope of this presentation. In Sec.2 we provide a brief synopsis of the heavy baryon s - and p -wave states expected to arise in a constituent quark model classification. In Sec.3 we provide a general formula which allows one to count the number of independent reduced amplitudes in current, pion and photon transitions between heavy baryons in the HQS limit. In Sec.4 we describe how the constituent quark model leads to a reduction in the number of the reduced amplitudes in heavy baryon transitions. We discuss some explicit examples and give results on the coupling complexity that remains after invoking the constituent quark model for the light-side diquark transitions. We limit our attention to transitions involving s - and p -wave states.

2 Heavy Baryon s - and p -Wave States

A heavy baryon is made up of a light diquark system (qq) and a heavy quark Q . The light diquark system has bosonic quantum numbers j^P with total angular momentum $j = 0, 1, 2, \dots$ and parity $P = \pm 1$. To each diquark system with spin-parity j^P there is a degenerate heavy baryon doublet with $J^P = (j \pm \frac{1}{2})^P$ ($j = 0$ is an exception). It is important to realize that the HQS structure of the heavy baryon states is entirely determined by the spin-parity j^P of the light diquark system. The requisite angular momentum coupling factors can be read off from the coupling scheme $J^P \otimes \frac{1}{2}^+ \Rightarrow J^P$. Apart from the angular momentum coupling factors the dynamics of the light system is completely decoupled from the heavy quark.

Next I turn my attention to the question of which low-lying heavy baryon states can be expected to exist. From our experience with light baryons and light mesons we know that one can get a reasonable description of the light particle spectrum in the constituent quark model picture. This is particularly true for the enumeration of states, their spins and their parities. As much as we know up to now, gluon degrees of freedom do not seem to contribute to the particle spectrum. It is thus quite natural to try the same constituent approach to enumerate the light diquark states, their spins and their parities.

From the spin degrees of freedom of the two light quarks one obtains a spin 0 and a

spin 1 state. The total orbital state of the diquark system is characterized by two angular degrees of freedom which I take to be the two independent relative momenta $k = \frac{1}{2}(p_1 - p_2)$ and $K = \frac{1}{2}(p_1 + p_2 - 2p_3)$ that can be formed from the two light quark momenta p_1 and p_2 and the heavy quark momentum p_3 . The k -orbital momentum describes relative orbital excitations of the two quarks, and the K -orbital momentum describes orbital excitations of the center of mass of the two light quarks relative to the heavy quark. The (k, K) -basis is quite convenient in as much as it allows one to classify the diquark states in terms of $SU(2N_f) \otimes O(3)$ representations[2]. In this presentation I limit my attention to the ground state s -wave and excited p -wave heavy baryon states as they occur in the constituent approach to the light diquark excitations. We specify to $N_f = 2$ for two (u, d) -flavours. The s -wave states $\Lambda_Q(1/2^+)$ and $\{\Sigma_Q(1/2^+, 3/2^+)\}$ are in the $10 \otimes 1$ representation of $SU(4) \otimes O(3)$. The K -type p -wave states are in the $10 \otimes 3$ representation with the particle content[2,3]

$$\{\Lambda_{QK1}^{**}(1/2^-, 3/2^-)\}, \Sigma_{QK0}^{**}(1/2^-), \{\Sigma_{QK1}^{**}(1/2^-, 3/2^-)\} \text{ and } \{\Sigma_{QK1}^{**}(3/2^-, 5/2^-)\}, \quad (1)$$

The k -type p -wave states, finally, are in the $6 \otimes 3$ representation with the particle content[2,3]

$$\{\Sigma_{Qk1}^{**}(1/2^-, 3/2^-)\}, \Lambda_{Qk0}^{**}(1/2^-), \{\Lambda_{Qk1}^{**}(1/2^-, 3/2^-)\} \text{ and } \{\Lambda_{Qk1}^{**}(3/2^-, 5/2^-)\}. \quad (2)$$

HQS doublets are written in curly brackets. Apart from the ground state s -wave baryons one thus has altogether seven Λ -type p -wave states and seven Σ -type p -wave states. This analysis can easily be extended to the case $SU(6) \otimes O(3)$ bringing in the strangeness quark in addition.

3 Generic Picture of Current, Pion and Photon Transitions

In $b \rightarrow c$ current transitions, and $c \rightarrow c$ pion and photon transitions between heavy baryons the heavy-side and light-side transitions occur completely independent of each other (they “factorize”) in the HQS limit except for the requirement that the heavy side

and the light side have the same velocity in the initial and final state, respectively, which are also the velocities of the initial and final heavy baryons. The three types of transitions are depicted in Fig.1. The $b \rightarrow c$ current transition induced by the flavour-spinor matrix Γ is hard and, accordingly, in general there is a change of velocities $v_1 \rightarrow v_2$, whereas there is no velocity change in the pion and photon transitions. The heavy-side transitions are completely specified whereas the light-side transitions $j_1^{P_1} \rightarrow j_2^{P_2}$, $j_1^{P_1} \rightarrow j_2^{P_2} + \pi$ and $j_1^{P_1} \rightarrow j_2^{P_2} + \gamma$ are described by a number of amplitudes which parametrize the light-side transitions. The pion and the photon couple only to the light side. In the case of the pion this is due to its flavour content. In the case of the photon the coupling of the photon to the heavy side involves a spin flip which is done by $1/m_Q$ and thus the photon couples only to the light side in the HQS limit.

The number N of independent reduced HQS amplitudes can be obtained by performing a light-side helicity or LS -amplitude analysis [3]. The result is

current transitions:

$$\begin{aligned} n_1 \cdot n_2 &= 1 & N &= j_{\min} + 1 \\ n_1 \cdot n_2 &= -1 & N &= j_{\min} \end{aligned} \tag{3}$$

pion transitions:

$$\begin{aligned} n_1 \cdot n_2 &= 1 & N &= j_{\min} \\ n_1 \cdot n_2 &= -1 & N &= j_{\min} + 1 \end{aligned} \tag{4}$$

photon transitions:

$$\begin{aligned} j_1 &= j_2 & N &= 2j_1 \\ j_1 &\neq j_2 & N &= 2j_{\min} + 1 \end{aligned} \tag{5}$$

In the case of current and pion transitions the counting involves the normalities of the light-side diquarks which are defined by $n = (-1)^j P$.

4 Constituent Quark Model Approach to Light-Side Transitions

Interest in the constituent quark model has recently been rekindled by the discovery (or rediscovery) that two-body spin-spin interactions between quarks are non-leading in $1/N_C$, at least in the baryon sector [1]. Thus, to leading order in $1/N_C$, light quarks behave as if they were heavy as concerns their spin interactions. In the constituent quark model approach one further assumes that spin and orbital degrees of freedom decouple. One can therefore classify the light diquark system in terms of $SU(2N_f) \otimes O(3)$ symmetry multiplets. Transitions between light quark systems are parametrized in terms of a set of one-body operators whose matrix elements are then evaluated between the $SU(2N_f) \otimes O(3)$ multiplets [2,4,5].

Let us illustrate this for the $(b \rightarrow c)$ current induced ground state to ground state transitions. For these there are altogether three reduced HQS form factors or Isgur-Wise functions, one for the $\Lambda_b \rightarrow \Lambda_c$ transition and two for the $\{\Sigma_b\} \rightarrow \{\Sigma_c\}$ transitions. The ground state to ground state transition is simple in that there is only one one-body operator. Thus the three reduced HQS form factors can all be expressed in terms of a single form factor $A(\omega)$, where $A(1) = 1$ at zero recoil. One then finds that the current transition amplitudes are given by [2,3,6]

$$\begin{aligned} \Lambda_b \rightarrow \Lambda_c \quad : \quad M^\lambda &= \bar{u}_2 \Gamma^\lambda u_1 \frac{\omega + 1}{2} A(\omega) \\ \{\Sigma_b\} \rightarrow \{\Sigma_c\} \quad : \quad M^\lambda &= \bar{\psi}_2^\nu \Gamma^\lambda \psi_1^\mu \left(-\frac{\omega + 1}{2} g_{\mu\nu} + \frac{1}{2} v_1^\nu v_2^\mu \right) A(\omega) \end{aligned} \tag{6}$$

The same result has been obtained by C.K.Chow by analyzing the large N_C limit of QCD [7].

For the current transitions from the bottom baryon ground states to the p -wave charm baryon states one similarly reduces the number of reduced form factors when using the constituent quark model in addition to HQS. For the transition into the K -multiplet one has a reduction from five HQS reduced form factors to two constituent quark model form factors whereas for transitions into the k -multiplet one can relate two HQS reduced form factors to one single spin-orbit form factor [2]. These are testable predictions in as

	HQS	constituent quark model
<i>Current transitions:</i>		
s -wave to s -wave	3	1
s -wave to p -wave (K)	5	2
s -wave to p -wave (k)	2	1
<i>Pion transitions:</i>		
s -wave to s -wave	2	1
p -wave (K) to s -wave	7	2
p -wave (k) to s -wave	5	2
<i>Photon transitions:</i>		
s -wave to s -wave	3	1
p -wave (K) to s -wave	11	2
p -wave (k) to s -wave	11	2

Table 1: Number of independent amplitudes for current, pion and photon transitions between heavy baryons in the HQS limit and in the constituent quark model

much as the population of helicity states in the daughter baryon is fixed resulting in a characteristic decay pattern of its subsequent decay.

The one-pion and photon transitions can be treated in a similar manner. Again one finds a significant simplification of the HQS structure, i.e. the number of independent HQS amplitudes is reduced from those listed in Eqs. (4) and (5) when the constituent quark model is invoked in addition to HQS. Results for the one-pion transitions can be found in [4] and for the photon transitions in [5]. We summarize our results in Table1 where we list the number of independent reduced HQS amplitudes together with the number of independent amplitudes that remain when the constituent quark model is invoked. We mention that the dramatic reduction in the number of independent amplitudes shown in Table1 is specific to the heavy baryon sector and is no longer true for heavy mesons.

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Figure Captions

Fig. 1: Generic picture of bottom to charm current transitions, and pion and photon transitions in the charm sector in the HQS limit $m_Q \rightarrow \infty$

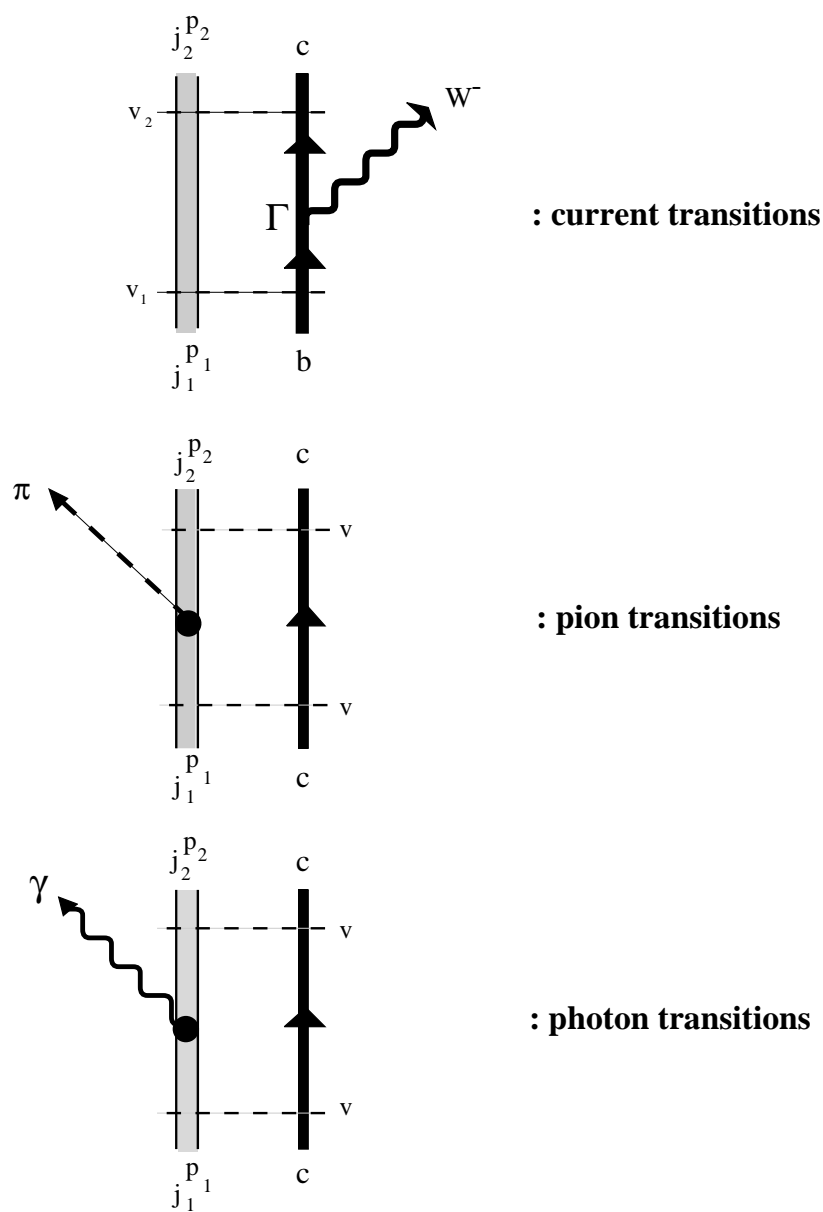


Figure 1